I guess the posts on Exponents and Roots have been lifeless for the most part. If I thought they were a drag to write (I hate rules!) , I am sure you thought they were a torture to read. Nevertheless, if you find yourself wondering what to do when you see (3^2 * 9^4), you need to go through them (can't live without the rules either!) The good news is that now that we are done with the basics, we can go on to the more "fun" concepts. And the best way to learn fun concepts is through fun questions. So let's take a couple of problems (found them on a GMAT forum) dealing with comparisons of exponents/roots.

Question 1: Which of the following represents the greatest value?

A) √2/√3 + √3/√4 + √4/√5 + √5/√6
B) 2/3 + 3/4 + 4/5 + 5/6
C) 2^2/3^2 + 3^2/4^2 + 4^2/5^2 + 5^2/6^2
D) 1 – 1/3 + 4/5 – 3/4
E) 1 – 3/4 + 4/5 + 1/3

Solution:

My first thought is to eliminate some options. I see options D and E have negative terms. A quick look tells me that they are definitely smaller than the other options.

Option D: 1 – 1/3 + 4/5 – 3/4

(1 – 1/3) is 2/3 and (4/5 – 3/4) is something very small too. The sum of these two terms is definitely much less than the sum obtained in option B where each term is a little less than 1. So option D cannot be the answer.

Option E: 1 – 3/4 + 4/5 + 1/3

(1 – 3/4) is 1/4

1/4 + 4/5 + 1/3 is definitely less than the sum obtained in option B. So option E cannot be the answer.

Option C: Each term of option B is squared in this option. Each term of option B is less than 1 so when you square it, it becomes even smaller (concept was discussed in this post). Hence every term of option C is smaller than every corresponding term of option B. Therefore, the sum obtained in option C will be less than the sum obtained in option B.

Now we are left with two options, A and B. Let's compare them. You know that we can easily compare fractions that have the same numerator or denominator. For example, out of 2/7 and 4/7, we know that 4/7 is greater because it has the greater numerator. Out of 4/9 and 4/5, we know that 4/5 is greater because it has the smaller denominator. How do you compare when both numerator and denominator are different? Simple – You need to make either their denominator or numerator equal.

Say I want to compare √2/√3 with 2/3.

I just multiply and divide √2/√3 by √3 to get √6/3.

Since √6, the numerator of √6/3, is greater than 2, the numerator of 2/3, we get that √2/√3 is greater than 2/3.

Similarly, all terms of option A will be greater than all corresponding terms of option B. Therefore, the sum obtained in option A will be more than the sum obtained in option B.

Answer (A).

Note: You can say intuitively that √2/√3 is greater than 2/3 because √2 is not much smaller than 2 and √3 is not much smaller than 3 but the difference between √2 and √3 is much smaller than the difference between 2 and 3. But you need to know your numbers very well to make such intuitive decisions correctly. If you have any doubts, just follow the approach of comparing by making the numerator/denominator equal.

Let's look at another example now. This one asks us to compare roots.

Question 2: Which of the following quantities is the largest?

(A) square root (2)
(B) cube root (3)
(C) fourth root (4)
(D) fifth root (5)
(E) sixth root (6)

Solution:

First, let's convert the roots to exponents.

We have to find the greatest out of: 2^(1/2), 3^(1/3), 4^(1/4), 5^(1/5) and 6^(1/6)

Since fractional powers are a pain, let us multiply all the powers by 60 (the LCM of 2, 3, 4, 5 and 6) to make them manageable. Note here that even though we have changed the numbers, we can still compare them. It is like saying that we have two numbers a and b, both positive and greater than 1. If we find out which of a^{60} and b^{60} is greater, we can find whether a is greater than b or not. The logic is that if $a > b$ (given a and b are greater than 1), then any positive integral power of a will be greater than the same power of b i.e. $a^{20} > b^{20}$, $a^{27} > b^{27}$, $a^{60} > b^{60}$ etc. We are using the same logic here.

The numbers now become: 2^{30}, 3^{20}, 4^{15}, 5^{12} and 6^{10}.

Let's compare these. First thing we notice is that $4^{15} = (2^2)15 = 2^{30}$. So option A and C have the same value. Since there is only one answer, we can be certain that it is not out of A and C. There must be another value greater than 2^{30}.

We are left with: 3^{20}, 5^{12} and 6^{10}.

Bases and powers, both are different in these three options. To compare, we need to make one of them the same. We see that $3^{20} = 9^{10}$. We can compare this with 6^{10}. We see that 9^{10} (i.e. 3^{20}) is greater.

Now we just need to compare 3^{20} with 5^{12}.

$3^{20} = (3^5)^4 = 243^4$

$5^{12} = 125^4$

Out of these two, 3^{20} is greater i.e. $3^{(1/3)}$ is the greatest of all the five options.

Answer (B)

A little bit of manipulation in both the questions led us quickly to the answers. Hope you enjoyed working on these problems. Keep practicing!